Critical currents in Josephson junctions with unconventional pairing symmetry: $d_{x^2-y^2}+is$ versus $d_{x^2-y^2}+id_{xy}$

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Phenomenological Ginzburg-Landau theory is used to calculate the possible spontaneous vortex states that may exist at corner junctions of $d_{x^2-y^2}+ix$-wave (where $x=s$ or $x=d_{xy}$) and s-wave superconductors. We study the magnetic flux and the critical current modulation with the junction orientation angle θ, the magnitude of the order parameter, and the magnetic field H. It is seen that the critical current I_c versus the magnetic flux Φ relation is symmetric/asymmetric for $x=d_{xy}/s$ when the orientation is exactly such that the lobes of the dominant $d_{x^2-y^2}$-wave order parameter points towards the two junctions, which are at right angles for the corner junction. The conclusion is that a measurement of the $I_c(\Phi)$ relation may distinguish which symmetry ($d_{x^2-y^2}+is$ or $d_{x^2-y^2}+id_{xy}$) the order parameter has.

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I. INTRODUCTION

One of the main questions in the research activity on high-T_c superconductors nowadays is the identification of the order parameter symmetry and its underlying mechanism.[1,2] The most possible scenario is that the bulk pairing state has a $d_{x^2-y^2}$-wave character. Theoretical calculations, suggest that an imaginary s-wave component which breaks the time reversal symmetry is induced in some cases, wherever the $d_{x^2-y^2}$-wave order parameter varies spatially such as near a vortex, or near the surface.[3] Also the observation of fractional vortices on a triangular grain boundary in $YBa_2Cu_3O_7$ by Kirtley *et al.*,[4] may indicate a possible violation of the time-reversal symmetry near grain boundary. Theoretical explanation of this experiment is given by Bailey *et al.* in Ref. 5 where they study a triangular grain boundary in d-wave superconductors. They conclude that under the assumption of d-wave symmetry, the flux at the edges of this triangle can take the values $\pm\Phi_0/2$, which does not agree with the experiment. However under the assumption of $d_{x^2-y^2}+is$-wave symmetry an intrinsic phase shift $\phi_c(x)$ exists in each triangle edge. In turn the phase $\phi(x)$ must change in order to connect the different values of ϕ_c in each segment. This arrangement leads to fractional vortices or antivortices at each three corners, in agreement with the experiment.

Another pairing state which breaks the time reversal symmetry is the $d_{x^2-y^2}+id_{xy}$ wave. Patches of complex d_{xy} components are induced around magnetic impurities at low temperatures in a $d_{x^2-y^2}$-wave superconductor forming a phase coherent state as a result of tunneling between different patches.[6] Violation of parity and time reversal symmetry occurs in this state. Also on the high field region, $H \leqslant H_{c2}$ the $d_{x^2-y^2}$-wave state can be perturbed by the external filed, producing a $d_{x^2-y^2}+id_{xy}$ state in the bulk.[7]

The observation of the splitting of the zero energy peak in the conductance spectra at low temperatures indicates that a secondary component is induced which violates locally the time reversal symmetry.[8] Theoretical explanation based on surface-induced Andreev states, has been proposed.[9] Recently the field dependence of this splitting has been observed in the tunneling spectra of YBCO.[10,11] This observation is consistent with a $d_{x^2-y^2}+is$ surface order parameter as well as a $d_{x^2-y^2}+id_{xy}$ bulk order parameter. Another question which can be asked is to what extend, the observation of a symmetric magnetic interference pattern in the corner junction experiments[2] is an identification of $d_{x^2-y^2}$-wave symmetry, or could also imply a $d_{x^2-y^2}+id_{xy}$ pairing state also? In this work we propose a phase sensitive experiment based on the Josephson effect, which may be used to distinguish which symmetry ($d_{x^2-y^2}+is$ or $d_{x^2-y^2}+id_{xy}$) the order parameter has near the surface. We study the static properties of a frustrated junction which is made of two one-dimensional junctions, of $d_{x^2-y^2}+ix$-wave (where $x=d_{xy}$ or $x=s$) and s-wave superconductors. By introducing an extra relative phase in one part of this junction, the above junction can be mapped into the corner junctions experiments.[2,12] We examine the spontaneous flux and the critical current modulation of the vortex states with the junction orientation angle θ, the magnitude of the secondary component n_s, and the magnetic field H. In each case we derive simple arguments which are useful to discriminate between the time reversal symmetry broken states. For example, when the orientation is exactly such that the lobes of the dominant $d_{x^2-y^2}$-wave order parameter points towards the junction interface the magnetic interference pattern is symmetric (asymmetric) when the secondary order parameter is $x=d_{xy}(s)$. This is verified for small junctions as well as in the long junction limit, and can be used to distinguish between broken time reversal symmetry states.

The rest of the paper is organized as follows. In Sec. II we discuss the Josephson effect between a superconductor with broken time reversal symmetry and an s-wave superconductor. In Sec. III the geometry of the corner junction is discussed. In Sec. IV we present the results for the magnetic flux of the spontaneous vortex states in corner junctions with some intrinsic magnetic flux. In Sec. V the parameters which can modulate the spontaneous flux and the critical currents are considered. In Sec. VI a connection with the experiment is made. Finally, a summary and discussion are presented in the last section.

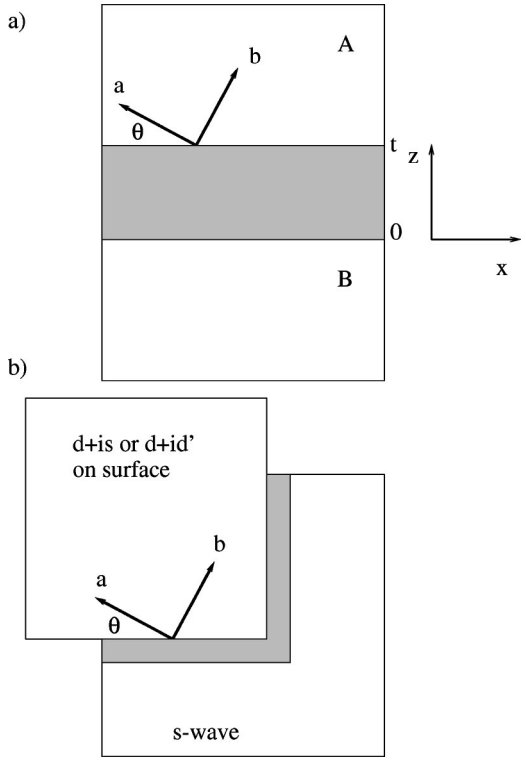

FIG. 1. (a) A single Josephson junction between superconductors A and B with a two component order parameter. The angle between the crystalline a axis of A and the junction interface is θ. (b) The geometry of the corner junction between a mixed symmetry superconductor, and an s-wave superconductor.

II. JOSEPHSON EFFECT BETWEEN TWO SUPERCONDUCTORS WITH MIXED WAVE SYMMETRY

We consider the junction shown in Fig. 1(a), where two superconductors (A in the region $z>t$ and B in the region $z<0$), are separated by an intermediate layer. We assume that each superconductor has a two component order parameter. The order parameter for each component $k(k=1,2)$ in the superconductors, can be written as

$$n_k=\begin{cases}\tilde{n}_k^A e^{i\phi_k^A}, & z>t,\\ \tilde{n}_k^B e^{i\phi_k^B}, & z<0.\end{cases} \tag{1}$$

Here $\phi_k^{A(B)}$ is the phase of the order parameter n_k in superconductor $A(B)$. Then phenomenological Ginzburg-Landau theory is used to calculate the supercurrent density given by[13]

$$J=\sum_{k,l=1}^{2} J_{ckl}\sin(\phi_k^B-\phi_l^A), \tag{2}$$

where

$$\begin{aligned} J_{c11}&=(2e\hbar/m_a^* d)\tilde{n}_1^A\tilde{n}_1^B,\\ J_{c21}&=(2e\hbar/m_\nu^* d)\tilde{n}_1^A\tilde{n}_2^B,\\ J_{c12}&=(2e\hbar/m_\nu^* d)\tilde{n}_2^A\tilde{n}_1^B,\\ J_{c22}&=(2e\hbar/m_b^* d)\tilde{n}_2^A\tilde{n}_2^B. \end{aligned} \tag{3}$$

m_a^*, m_ν^*, m_b^* are the effective masses that enter into the Ginzburg-Landau equations. In the following these masses are taken equal to an effective mass m^*.

We restrict to the case where B is an s wave. In this case $\tilde{n}_1^B=0$, and $\tilde{n}_2^B=\text{const}$. We define $\phi=\phi_2^B-\phi_1^A$, as the relative phase difference between the two superconductors. We consider the case where the intrinsic phase difference within superconductor A is $\phi_2^A-\phi_1^A=\pi/2$. Then the order parameter in A is complex and breaks the time reversal symmetry. The supercurrent density can be written as

$$J(\phi)=\tilde{J}_c\sin(\phi+\phi_c), \tag{4}$$

with

$$\tilde{J}_c=\sqrt{J_1^2+J_2^2}, \tag{5}$$

$$\phi_c=\begin{cases}\tan^{-1}\dfrac{J_2}{J_1}, & J_1>0,\\ \pi+\tan^{-1}\dfrac{J_2}{J_1}, & J_1<0,\end{cases} \tag{6}$$

where $J_1=J_{c21}$, $J_2=-J_{c22}$. The Josephson critical current density $\tilde{J}_c$ is scaled in units of $J_{c0}=e\hbar/m^*d$. Two special cases are the following:

(i) For $d_{x^2-y^2}+is$-wave case the magnitude of the $d_{x^2-y^2}$-wave component in Eq. (1) is $\tilde{n}_1^A=n_{10}\cos(2\theta)$, where θ is the angle of the crystalline a axis of superconductor A with the junction interface. The magnitude of the secondary order parameter in superconductor A is $\tilde{n}_2^A=n_{20}=0.1n_{10}$.

(ii) For $d_{x^2-y^2}+id_{xy}$-wave case, the magnitude of the $d_{x^2-y^2}$-wave component in Eq. (1) is given by $\tilde{n}_1^A=n_{10}\cos(2\theta)$, while the d_{xy} wave component is $\tilde{n}_2^A=n_{20}\sin(2\theta)$, where $n_{20}=0.1n_{10}$. This order parameter can occur in the following way: The order parameter magnitude for the d-wave state $\Delta_0(\theta)=\Delta_0\cos(2\theta)$ is an equal admixture of pairs with orbital moment $L_z=\pm 2$, and can be written as $\Delta_0(\theta)=(\Delta_0/2)[\exp(2i\theta)+\exp(-2i\theta)]$. In the presence of perturbation such as (ferromagnetically) ordered impurity spins S_z the coefficients of $L_z=\pm 2$ components will shift linearly in S_z with opposite signs. The final state will be $\Delta_0(\theta)\rightarrow\Delta_0(\theta)+iS_z\Delta_1(\theta)$, where $\Delta_1(\theta)=\sin(2\theta)$. The strength of the secondary component is proportional to the perturbation S_z.

III. THE CORNER JUNCTION GEOMETRY

We consider the corner junction shown in Fig. 1(b), between a superconductor with broken time reversal symmetry

at the surface and an s-wave superconductor. If the angle of the a axis with the interface in the x direction is θ, then the corresponding angle in the z direction will be $\pi/2-\theta$. We map the two segments each of length $L/2$ where $L=10\lambda_J$ of this junction into a one-dimensional axis. In this case the two-dimensional junction can be considered as being made of two one dimensional junctions described in Sec. II connected in parallel. Their characteristic phases ϕ_{c1} and ϕ_{c2} depend upon the angle θ. We call this junction frustrated since the two segments have different characteristic phases ϕ_{c1},ϕ_{c2}. The fabrication details of corner junctions or superconducting quantum interference device (SQUID), between sample faces at different angles can be found in Refs. 2,12.

The superconducting phase difference ϕ across the junction is then the solution of the sine-Gordon (sG) equation

$$\frac{d^2\phi(x)}{dx^2}=\tilde{J}_c\sin[\phi(x)+\phi_c(x)]-I^{ov}, \tag{7}$$

with the boundary conditions

$$\left.\frac{d\phi}{dx}\right|_{x=0,L}=H. \tag{8}$$

The length x is scaled in units of the the Josephson penetration depth given by

$$\lambda_J=\sqrt{\frac{\hbar c^2}{8\pi e d J_{c0}}},$$

where d is the sum of the s wave, and mixed wave λ_{ab} penetration depths plus the thickness of the insulator layer. The relative phase $\phi_c(x)$ is $\phi_{c1}(\phi_{c2})$ in the left (right) part of the junction. The external magnetic field H, scaled in units of $H_c=\hbar c/2ed\lambda_J$ is applied in the y direction, which is considered small compared to λ_J. The bias current per unit length I^{ov} in the overlap geometry is scaled in units of $(c/4\pi)H_c$, and is uniformly distributed along the entire x axis of the junction. We can classify the different solutions obtained from Eq. (7) with their magnetic flux content

$$\Phi=\frac{1}{2\pi}(\phi_R-\phi_L), \tag{9}$$

where $\phi_{R(L)}$ is the value of ϕ at the right (left) edge of the junction, in units of the flux quantum $\Phi_0=hc/2e$.

IV. SPONTANEOUS VORTEX STATES

First let us consider the case where the two one-dimensional junctions of $d_{x^2-y^2}+ix$ wave where $x=s$ or $x=d_{xy}$, and s-wave supeconductors, each of length $L/2$, described in Sec. II are uncoupled. Then for $0<x<L/2$ the stable solutions for the sG equation are $\phi(x)=-\phi_{c1}+2n_1\pi$, where $n_1=0,\pm1,\pm2,\ldots$, while for $L/2<x<L$ the stable solutions for the sG equation are $\phi(x)=-\phi_{c2}+2n_2\pi$, where $n_2=0,\pm1,\pm2,\ldots$, where ϕ_{c1}, ϕ_{c2}, are the relative phases in each part of the junction due to different orientations. These solutions are plotted in Fig. 2, for $n_i=0,-1$, $i=1,2$ as a function of the orientation angle θ. When the frustrated junction is formed, and we consider the above junctions in parallel, the phase ϕ is forced to change around $x=L/2$, to connect these stable solutions. This variation of the phase ϕ, along the junction describes the Josephson vortices. The flux content of these states (in units of Φ_0) is[14]

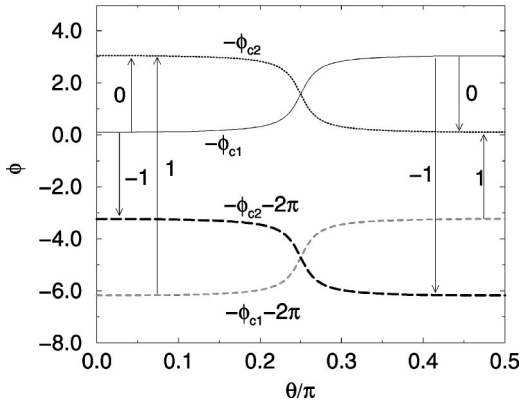

FIG. 2. The stable solutions $\phi=-\phi_{c1}+2n_1\pi$ ($\phi=-\phi_{c2}+2n_2\pi$), for $n_i=0,-1$, $i=1,2$, that exist in the left (right) junction, of $d_{x^2-y^2}+is$-wave and s-wave superconductors, when considered uncoupled, at zero current, versus the orientation angle θ. Each junction has length $L/2$, where $L=10\lambda_J$, and $\phi_{c1},(\phi_{c2})$ is the extra phase difference in the left (right) junction due to the different orientations. The arrows denote the variation of the phase ϕ in order to connect these stable solutions in the frustrated junction geometry. We present three possible solutions, i.e., $n=0$, $-1,1$, and down (up) arrow denotes negative (positive) magnetic flux.

$$\Phi=[\phi(L)-\phi(0)]/2\pi=(-\phi_{c2}+\phi_{c1}+2n\pi)/2\pi, \tag{10}$$

where the n value ($n=n_1-n_2=0,\pm1,\pm2,\ldots$) distinguishes between solutions with different flux content. We will concentrate to solutions called modes with the minimum flux content i.e., $n=0,1,-1$. Their magnetic flux in terms of ϕ_{c1},ϕ_{c2} is shown in Table I. Generally the flux content is fractional, i.e., is neither integer nor half-integer, as a consequence of the broken time reversal symmetry of the problem.

In the actual numerical simulations, the stable solutions of the sine-Gordon equation in the left (right) part of the junc-

TABLE I. The magnetic flux (Φ) in terms of ϕ_{c1}, ϕ_{c2} for the spontaneous solutions that exist in the corner junction geometry between a superconductor with time reversal broken symmetry and an s-wave superconductor (ϕ_{c1},ϕ_{c2} is the extra phase difference in the two edges of the corner junction due to the different orientations, of the a axis of the dominant $d_{x^2-y^2}$-wave superconductor). We present only the minimum flux states $n=0,-1,1$.

Vortex state n	Magnetic flux (Φ)
0	$(-\phi_{c2}+\phi_{c1})/2\pi$
1	$(-\phi_{c2}+\phi_{c1}+2\pi)/2\pi$
-1	$(-\phi_{c2}+\phi_{c1}-2\pi)/2\pi$

tion are taken as the initial conditions for the iteration procedure. For example for the $n=0$ solution the phase $\phi(x)$ is taken $\phi(x)=-\phi_{c1}$ $(-\phi_{c2})$ in the left (right) part of the junction, as an initial condition and then is iterated until convergence. In addition, if we take as initial condition , $\phi(x)=-\phi_{c1}$, in the left side, and $\phi(x)=-2\pi-\phi_{c2}$ in the right side, the final state of the system, after the iteration procedure, is the solution which we call $n=-1$, with negative magnetic flux, and not exactly opposite to $n=0$. We comment here that the solutions after the iteration procedure have smooth variation as a function of the position, as opposed to the step function variation of the initial conditions.

For the $0-0$ junction $\phi_{c1}=\phi_{c2}=0$, and the flux becomes $\Phi=n$, so we say that the flux is quantized in integer units of Φ_0. In this case, there exist solutions with flux $\Phi=\cdots,-1,0,1,\cdots$.[15] These solutions, when $n\neq 0$ are stabilized by the application of an external magnetic field. In the case of a junction with some spontaneous flux, at least for the modes with lower flux content, the external field is not necessary since the spontaneous magnetization state is stable.

In the case of $0-\pi$ junction, where the intrinsic phase in the right (left) part of the junction is $\phi_{c2}=-\pi$ $(\phi_{c1}=0)$, the stable solutions of the sG equation are $\phi(x)=2n\pi$ for the left part, while $\phi(x)=\pi(2n+1)$ for the right part of the junction. In this case a $0-\pi$ junction is formed. The corresponding flux becomes $\Phi=(n+1/2)\pi$, and the particular values of $n=0$, $n=-1$ give the half vortex and antivortex solutions, with opposite fluxon content, $\Phi=0.5$ and $\Phi=-0.5$, respectively.

V. MAGNETIC FLUX AND CRITICAL CURRENT MODULATION

In the following we will describe three parameters which can alter the spontaneous flux and the critical currents of the vortex states described in the previous section, in a corner junction between a superconductor with time reversal broken symmetry and an *s*-wave superconductor. These include the orientation angle θ, the magnitude of the secondary order parameter n_s, and the magnetic field H. In each parameter separately we will point out the differences between the $d_{x^2-y^2}+is$, and $d_{x^2-y^2}+id_{xy}$ waves.

A. Junction orientation

For the $d_{x^2-y^2}+is$-wave case, we consider first the situation where θ is varied from 0 to $\pi/2$. In Fig. 3 we plot the spontaneous magnetic flux versus the angle (θ) for the different modes $n=0,-1,1$ in the corner junction geometry. As we can see the magnetic flux changes with orientation. For angle θ close to 0 or $\pi/2$ the spontaneous modes existing at $H=0$ are separated by an integer value of the magnetic flux. This is also the case in the pure *s*-wave superconductor junction problem. The difference is that the modes are found displaced to fractional values of magnetic flux, contrary to the *s*-wave case where the magnetic flux takes on integer values at $H=0$. In particular the vortex solution in the $n=0$ mode (solid line) contains less than half a fluxon for $\theta=0$, and as we increase the angle θ towards $\pi/4$ it continuously reduces its flux, i.e., it becomes flat exactly at $\theta=\pi/4$ and then it reverses its sign and becomes an antivortex with exactly opposite flux content at $\theta=\pi/2$ from that at $\theta=0$. In addition we have plotted in Fig. 4(a) the phase dis-

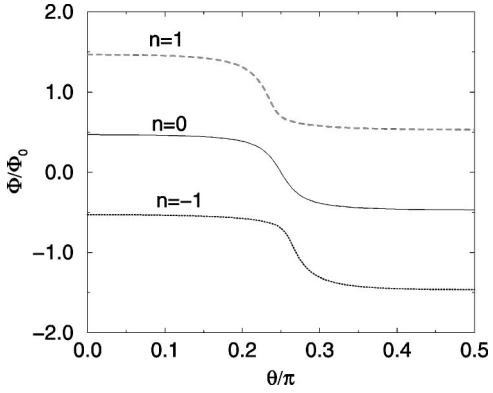

FIG. 3. The magnetic flux Φ as a function of the angle θ, for the various vortex states, $n=0,-1,1$, that exist spontaneously in a corner junction between a $d_{x^2-y^2}+is$-wave and an *s*-wave superconductor, with length $L=10\lambda_J$. The flux for $\theta=0$ is fractional.

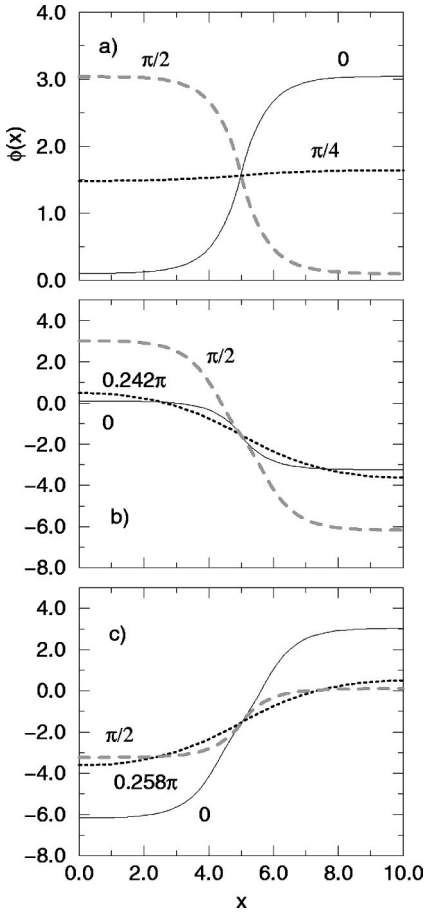

FIG. 4. The phase distribution of the vortex solutions (a) $n=0$, at $\theta=0$, $\pi/4$, $\pi/2$; (b) $n=-1$, at $\theta=0$, 0.242π, where the instability sets in, and $\pi/2$; (c) $n=1$, at $\theta=0$, 0.258π, at the point where the instability occurs, and $\pi/2$, for a corner junction of $d_{x^2-y^2}+is$-wave and *s*-wave superconductors, with length $L=10\lambda_J$, and zero overlap external current $I^{ov}=0$.

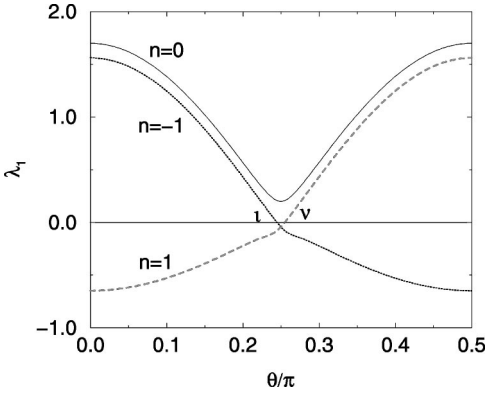

FIG. 5. The lowest eigenvalue λ_1 of the linearized eigenvalue problem as a function of angle θ, for the $n=0,-1,1$ solutions. In the range where θ is close to zero, the eigenvalues for both $n=0$, and -1 are positive and correspond to stable solutions.

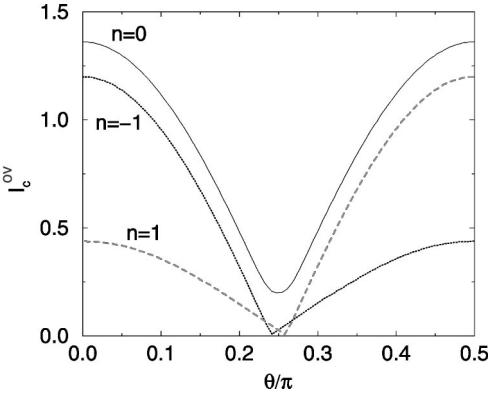

FIG. 6. Overlap critical current I_c^{ov} per unit length versus the angle θ for a corner junction of $d_{x^2-y^2}+is$-wave and s-wave superconductors, with length $L=10\lambda_J$, for the vortex solutions $n=0$, $-1,1$ that exist spontaneously in the junction.

tributions for the mode $n=0$ in different orientations $\theta=0,\pi/4,\pi/2$. The transition form the vortex to the antivortex mode as the orientation changes is clearly seen in this figure. Note that the solutions in this mode remain stable for all junction orientations. This is seen in Fig. 5 where we plot the lowest eigenvalue (λ_1) of the linearized eigenvalue problem as a function of the angle θ.[15] We see that $\lambda_1>0$, denoting stability for all values of the angle θ in this mode.

Let as now examine the solution in the $n=-1$ mode (dotted line in Fig. 3). We see that at $\theta=0$ it has negative flux, which in absolute value is more than $\Phi_0/2$ and as we increase the angle θ it decreases its flux to a full antifluxon when the orientation is slightly greater than $\pi/4$ and than to flux greater than Φ_0 when θ reaches $\pi/2$. As seen in Fig. 5 this solution becomes unstable at a point to the left of $\theta=\pi/4$ (point ι) due to the abrupt change of flux at this angle. More strictly the instability sets in due to the competition between the slope of the phase at the edges of the junction and at the junction center as the angle θ approaches the value $\pi/4$. At this point the slope competition makes the antivortex unstable. This is seen in Fig. 4(b) (dotted line) where the phase distribution for the $n=-1$ mode solution is plotted at the point where the instability starts, i.e., $\theta=0.242\pi$.

Finally the solution in the $n=1$ mode contains more than one fluxon at $\theta=0$ and is clearly unstable. It becomes stable at an angle slightly on the right of $\theta=\pi/4$ (point ν in Fig. 5) where the flux varies more smoothly, see $\theta=0.258\pi$ in Fig. 4(c). At $\theta=\pi/2$ it contains more than $\Phi_0/2$ in flux. We expect a time reversal broken symmetry state such as $d_{x^2-y^2}+is$ to be characterized by either the solution in the fractional vortex or antivortex mode, because due to the different character of these solutions a change from one variant to the other would demand the application of an external current or magnetic field and in this sense it would cost additional energy. So since these states are stable in external perturbations, once the system is prepared in one of these it will remain to that state.

In general we see that for each value of θ there exist in the junction a pair of stable solutions which when applying an external bias current will lead to observable critical currents. In Fig. 6 we plot the overlap critical current per unit length I_c^{ov} as a function of θ, at $H=0$, for the $n=0,-1,1$-mode solutions, in the $d_{x^2-y^2}+is$-wave case. In the overlap geometry the current is distributed in the entire x axis. In the calculations we have taken into account that the Josephson critical current density $\tilde{J}_c$ has a characteristic variation with the orientation. We find that for a given orientation it is possible for the junction current density to vary in the way that several modes with different critical currents can exist. In Fig. 7 we plot the current density when the total current is maximum, for different modes, and orientations, which will give us information about the actual shapes of the vortices. Let us consider the situation where the junction contains a solution in the mode $n=0$, at $\theta=0$, when the net current is maximum. The spatial variation of ϕ is described by a fractional vortex which is displaced around the value $\phi=\pi$, from the corresponding distribution at zero current which is around $\pi/2$ [see Fig. 4(a)]. The current density distribution as seen in Fig. 7(a) (solid line) at the maximum current is flat above unit with a small variation around the junction center giving rise to the large value on the net current, seen in Fig. 6. Also at $\theta=\pi/4$ the flat phase distribution corresponding to the $n=0$ solution at zero current is displaced towards the value $\phi=\pi$ when applying an external current. The corresponding current distribution seen in Fig. 7(a) (dotted line) is straight line and the net current is small for this orientation. For the $n=-1$ solution at the point where the instability sets in, i.e., $\theta=0.242\pi$, the current density distribution is symmetric around zero as seen in Fig. 7(b) (dotted line) and carries zero net current at this point. Thus the instability occurs just before the angle where a full antifluxon enters the junction. A slightly different situation occurs in the magnetic interference pattern of a pure s-wave superconductor junction[16] where, the net current is zero at the magnetic field where a full fluxon or antiluxon enters the junction, in the no flux 0 mode. At the point $\theta=\pi/2$, of the $n=-1$ mode the junction contains more than one fluxon causing the characteristic oscillations in the current density around the junction center as seen in Fig. 7(b) (dashed line). This reduces the critical current for this orientation.

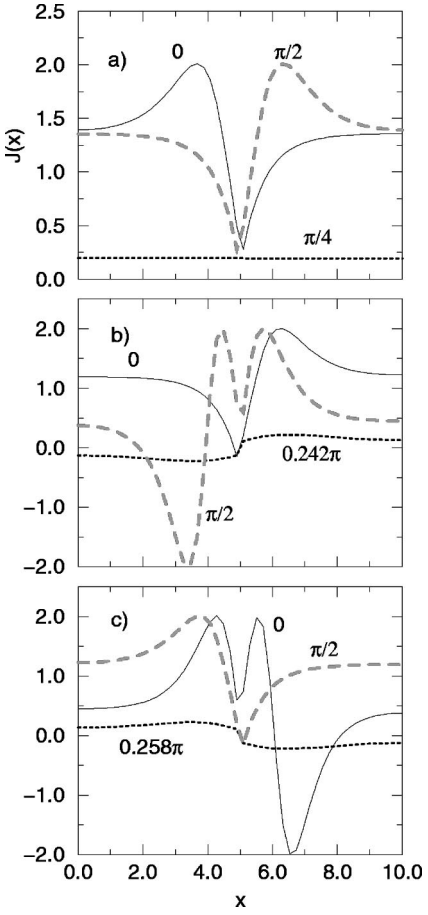

FIG. 7. The current density distribution $J(x)$ of the vortex solutions (a) $n=0$, at $\theta=0$, $\pi/4$, $\pi/2$; (b) $n=-1$, at $\theta=0$, 0.242π, where the instability sets in, and $\pi/2$; (c) $n=1$, at $\theta=0$, 0.258π, at the point where the instability occurs, and $\pi/2$, for a corner junction of $d_{x^2-y^2}+is$-wave and s-wave superconductors, with length $L=10\lambda_J$, and maximum external overlap current I_c^{ov}.

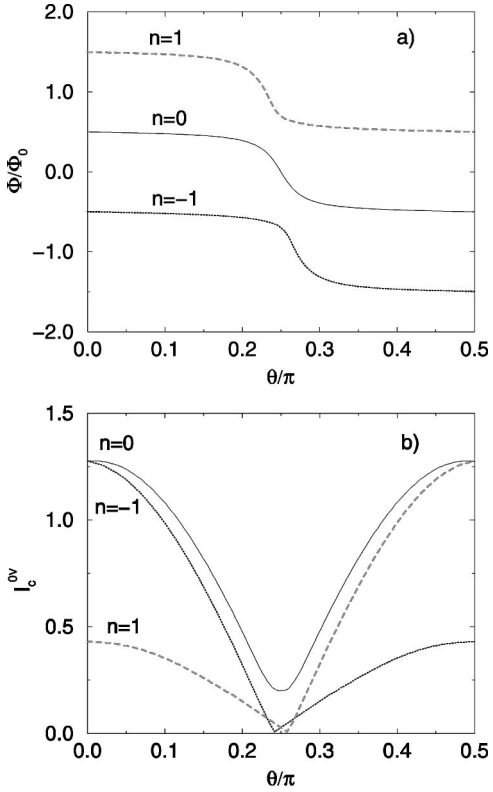

FIG. 8. (a) The spontaneous magnetic flux Φ as a function of the angle θ, for the various vortex states, $n=0,-1,1$, for a corner junction between a $d_{x^2-y^2}+id_{xy}$-wave and an s-wave superconductor, with length $L=10\lambda_J$. The flux for $\theta=0$ is integer multiply of $\Phi_0/2$. (b) The corresponding critical current I_c^{ov} per unit length.

For the $d_{x^2-y^2}+id_{xy}$ pairing symmetry state, we plot in Fig. 8(a) the flux content for the $n=0,-1,1$, versus the angle θ. Note the half integer or multiplies value of Φ at θ close to 0 or $\pi/2$. For this grain orientation the magnetic flux is only sensitive to the real part of the order parameter, which has a sign change but does not break time-reversal symmetry. In the $d_{x^2-y^2}+is$-wave state the order parameter is complex for all junction orientations and breaks the time-reversal symmetry. Close to 0 or $\pi/2$ the flux is fractional. The flux quantization at $\theta=0$ can be used to discriminate between these states.

In Fig. 8(b) we plot the critical current per unit length evolution with the grain angle θ in the $d_{x^2-y^2}+id_{xy}$-wave state. Close to $\theta=0$ we see that the I_c^{ov} for the $n=0,-1$ solutions, coincide. This happens also at $\theta=\pi/2$ for the $n=0,1$ solutions. In these orientations the order parameter becomes pure real and does not break the time-reversal symmetry. As a result the critical current at these angles is the same as in a junction with pure d-wave symmetry. At $\theta=\pi/4$ the order parameter is pure imaginary and has the same magnitude for both pairing states. As a consequence for $\theta=\pi/4$, the critical currents for both junctions coincide. Also the unstable part of the $n=1$ branch, in the I_c vs θ is almost the same for the two symmetry states, due to the small difference in the flux, compared with the large flux content of the solutions in this region.

B. Magnitude of the secondary order parameter

In the above calculations the magnitude of the secondary order parameter is small compared to the dominant (i.e., $n_{20}=0.1n_{10}$). However, the maximum fraction of the secondary component that has been observed in phase coherent experiments employing different materials, geometries, and techniques is up to 25% of the dominant.[2] This triggered our interest to study the magnetic flux and also the critical currents as a function of the strength (n_s) of the secondary order parameter, where the magnitude of the dominant order parameter n_d is also varied in a way that $n_s+n_d=1$. When $n_s=0$ only the $d_{x^2-y^2}$-wave order parameter is present, while when $n_s=1$ only the s-wave order parameter appears. This situation can be realized, for example, near the surface where the $d_{x^2-y^2}$-wave order parameter is suppressed and the s-wave order parameter is enhanced. The result is presented in Fig. 9(a) and 9(b) for the $d_{x^2-y^2}+is$-wave case at $\theta=0$. We see that when the secondary component is absent (i.e.,

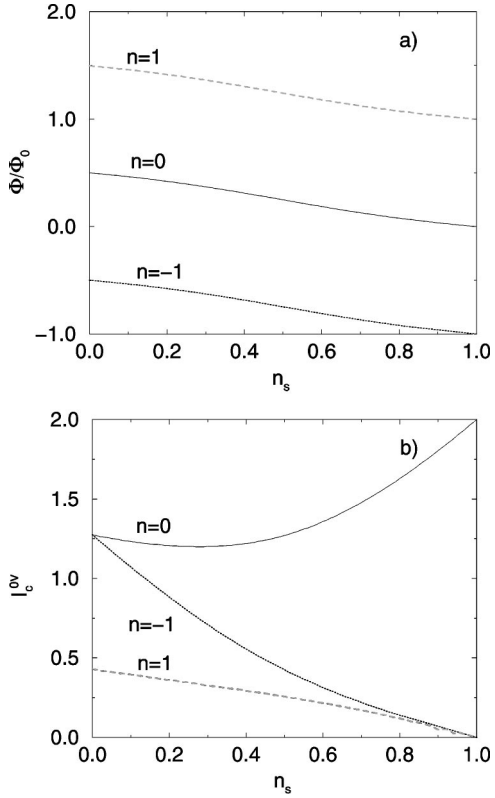

FIG. 9. (a) The spontaneous magnetic flux Φ and (b) the critical current I_c^{ov} per unit length versus the strength n_s of the secondary s-wave component for a corner junction of $d_{x^2-y^2}+is$-wave and s-wave superconductors, with length $L=10\lambda_J$, for the vortex solutions $n=0,-1,1$ that exist spontaneously in the junction. The magnitude n_d of the $d_{x^2-y^2}$-wave order parameter is given by the relation $n_s+n_d=1$.

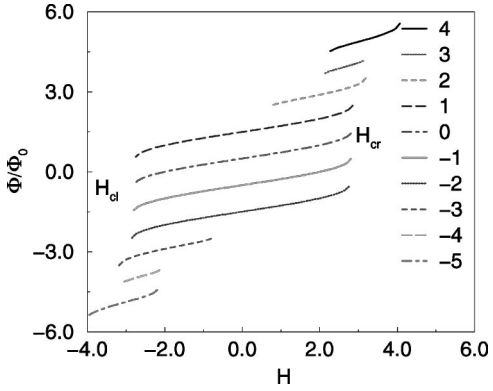

FIG. 10. Magnetic flux Φ/Φ_0 at zero external current versus the magnetic field H for a corner junction of $d_{x^2-y^2}+id_{xy}$-wave and s-wave superconductors, with length $L=10\lambda_J$, for angle $\theta=0°$. $H_{cl}(H_{cr})$ denotes the critical values of the magnetic field where the mode $n=0$, terminates.

$n_s=0$) the picture of the $d_{x^2-y^2}$-wave state is reproduced. The same picture also holds for the $d_{x^2-y^2}+id_{xy}$-wave state at $\theta=0$, since the order parameter for the $d_{x^2-y^2}+id_{xy}$-wave state at $\theta=0$ is real not breaking the time-reversal symmetry. So for $\theta=0$ the magnetic flux and the critical current for the $d_{x^2-y^2}+id_{xy}$-wave state would not show any change with the variation of the secondary order parameter d_{xy}. As n_s is increasing the modes $n=0$ and $n=-1$ are no more degenerate, in the sense that their flux deviates from the value $\Phi_0/2$ and $-\Phi_0/2$, respectively, and also their critical currents are no longer equal. The mode $n=0$ has larger critical current because it has smaller flux content in absolute value. For values of n_s close to unity, the different modes contain integer magnetic flux, as in the junction between s-wave superconductors, and also their critical currents have the same values as in the perfect junction problem. The conclusion is that the larger the secondary component is in a sample the easier is to be detected in a flux measurement experiment.

C. Magnetic field

We now examine the influence of the magnetic field on the spontaneous vortices for broken time reversal symmetry pairing states. In Fig. 10 we plot the magnetic flux at zero current versus the magnetic field H for the $d_{x^2-y^2}+id_{xy}$-wave state at $\theta=0$. In the pure s-wave superconductor junction there is no overlap between different modes in the magnetic flux, and each mode has magnetic flux which is more than $n\Phi_0$ and less than $(n+1)\Phi_0$. In this problem due to spontaneous magnetization the range of the modes is different and in some cases overlapping, and the labeling is with a single index n, corresponding to the pure s-wave superconductor junction (n,n+1) mode.[16] Moreover the range in magnetic flux of each mode is displaced compared to the pure s-wave superconductor junction problem by an amount which corresponds to the intrinsic flux. Also we have the existence of stable vortex states, i.e., $n=0,-1$, together with the unstable ones, i.e., $n=1$, -2 in a large interval of the magnetic field, which is almost the same. The $n=-2$ mode extends to zero magnetic field, and the reason we did not examined this mode in Sec. IV is because the stability analysis shows negative eigenvalues for all the range of junction orientations, at $H=0$. In the long s-wave junction the extremum of the mode (0,1) in H is the critical field for one fluxon (antifluxon) penetration from the edges [denoted by H_{cr} (H_{cl}), for the right (left) edge], and is equal to $2(-2)$. The solution for the phase at these extremum values of the field becomes unstable because the value of the phase at the junction edges reaches a critical value. In the problem of a junction with some spontaneous flux, we consider here, the range of the corresponding mode 0 in H is significantly broadened and also the instability at the boundaries sets in due to different reasons. In particular the instability occurs due to the interaction of the flux entering from the junction edges, when the magnetic field reaches the critical value $H_{cr}(H_{cl})$, with the spontaneous flux at the center. Similar features are encountered in the problem of flux pinning from a macroscopic defect in a conventional s-wave junction.[17]

We now examine the magnetic-interference pattern for the two symmetries where the bias current enters in the overlap geometry. In the $d_{x^2-y^2}+id_{xy}$-wave case, where $\theta=0$, this pattern has a symmetric form as we can see from Fig.

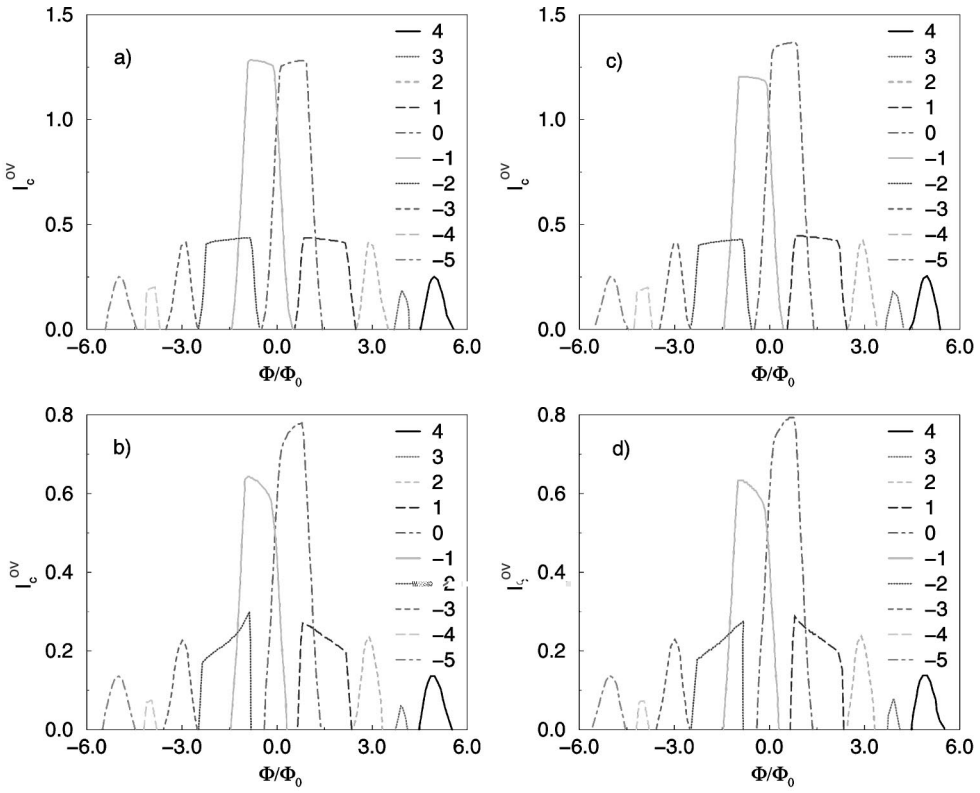

FIG. 11. (a) Overlap critical current I_c^{ov} per unit length versus the magnetic flux Φ in units of Φ_0, for a corner junction of $d_{x^2-y^2}+id_{xy}$-wave and *s*-wave superconductors, with length $L=10\lambda_J$, for angle $\theta=0°$. (b) The same as in (a) but for $\theta=0.5$. (c) The same as in (a) but for $d_{x^2-y^2}+is$-wave and *s*-wave superconductors for angle $\theta=0°$. (d) The same as in (c) but for $\theta=0.5$.

11(a). This is because this result is only sensitive to the real part of the order parameter, which has a sign change but does not break time-reversal symmetry. For the angle $\theta=0.5$ where the order parameter has a finite imaginary part and breaks the time-reversal symmetry this pattern becomes asymmetric and the ''dip'' appears to a value of flux slightly different than zero. Note that the asymmetry refers mainly to the modes $n=0$, and $n=-1$. The other modes are not influenced much due to their higher flux content. Also the critical current is suppressed compared to the case where $\theta=0$ as can be seen in Fig. 11(b), due to a drop in J_c.

In the $d_{x^2-y^2}+is$-wave symmetry, in the limit where $\theta\rightarrow 0$, the order parameter is complex and the pattern is asymmetric as can be seen in Fig. 11(c), for the angle $\theta=0$. This is in agreement with our previous work for the inline current input for a junction with $d_{x^2-y^2}+is$ symmetry.[18] There it was found that the pattern is asymmetric for lengths as long as $L=10\lambda_J$. For angles close to $\pi/4$, the magnetic interference pattern is similar with the $d_{x^2-y^2}+id_{xy}$ state. This is because the $\sin(2\theta)$ dependence of the d_{xy} component is almost unity. This is seen in Fig. 11(d) where we present the variation of the critical current per unit length versus the enclosed flux for $\theta=0.5$, and the symmetry state is $d_{x^2-y^2}+is$.

In the short junction limit $L<\lambda_J$ the same argument can be applied without any explicit reference to fractional vortex and antivortex solutions. However, as we found in our previous work,[18] both $n=0$ and $n=-1$ (there f_{va}, f_a) exist, with reduced flux content, in this limit as a continuation of the corresponding solutions in the large junction limit. In this case the external applied magnetic field becomes equal to the self-field, and the maximum current can be calculated analytically,[13]

$$\frac{I_m(\Phi)}{I_{m0}}=\left|\frac{\sin(\pi\Phi/2\Phi_0)\cos[\pi\Phi/2\Phi_0+(\phi_{c2}-\phi_{c1})/2]}{\pi\Phi/2\Phi_0}\right|. \tag{11}$$

As we see at $\theta=0$ for the $d_{x^2-y^2}+id_{xy}$-wave case, the relation $\phi_{c2}-\phi_{c1}=n\pi$ holds, and the magnetic interference pattern becomes symmetric, while for the $d_{x^2-y^2}+is$, this difference is a fraction of π and the pattern is asymmetric. However, as we increase the junction length, we expect this symmetric pattern for the *d*-wave order parameter to be continued. This symmetry in the large junction limit, is described more effectively by the assumption of the $n=0,-1$ solutions which give a symmetric magnetic interference pattern as we presented. Also the $n=-1$ solution extends to values for the magnetic flux, where the $n=0$ solution is absent. Eliminating one of them will break the symmetry of the diagram.

VI. EXPERIMENTAL RELEVANCE

The symmetric pattern with a minimum at zero applied field observed in corner junction experiments between YBCO and Pb at $\theta=0$ has been interpreted as an indication of $d_{x^2-y^2}$-wave symmetry.[19,20] This result refers to short junctions where the junction size is much smaller than the Josephson penetration depth. However, as we found here these experimental data are also consistent with an order parameter with $d_{x^2-y^2}+id_{xy}$ pairing symmetry at $\theta=0$.

Also the critical current I_c versus the magnetic flux Φ of a SQUID, consisting of two planar Josephson junctions on the faces of YBCO superconducting crystal, connected by a loop of a second superconductor, for $\theta=0$ or $\theta=\pi/2$ is

found shifted by $\Phi=0.5\Phi_0$ and has a minimum at $\Phi=0$ (instead of a maximum as in a SQUID involving conventional *s*-wave superconductors or the edge SQUID in which both junctions are on the same crystal face) but is still symmetric. This result has been attributed to an order parameter with $d_{x^2-y^2}$-wave symmetry. However the theoretical analysis done by Beasley *et al.*[21] shows that it is also consistent with an order parameter with $d_{x^2-y^2}+id_{xy}$ pairing symmetry at $\theta=0$ ($\theta=0$ corresponds to $\theta=\pi/2$ in the geometry of Ref. 21).

In both cases of SQUID and corner junction the symmetric pattern observed at $\theta=0$ rules out the $d_{x^2-y^2}+is$-wave pairing state where the order parameter is complex everywhere resulting in an asymmetric I_c versus Φ pattern for all angles θ. However, the small asymmetry (less than 2%) observed at $\theta=0$ in some experiments can be attributed to various complicating factors, e.g., fluxon trapping as will be discussed latter in this section.

The experiment proposed here to resolve ambiguity between $d_{x^2-y^2}+id_{xy}$ and $d_{x^2-y^2}$ at $\theta=0$, is to execute the same experiments using SQUID or corner junction at an angle between sample faces θ between 0 and $\pi/2$. Our calculation predicts symmetric (asymmetric) pattern for the $d_{x^2-y^2}$-wave ($d_{x^2-y^2}+id_{xy}$-wave) pairing state for the corner junction case. This kind of experiment has already been done in the case of SQUID geometry.[12] The tunneling directions are defined lithographically and patterned by ion milling of a *c*-axis oriented film. A YBCO thin film is patterned into a circle with a series of Nb-Au-YBCO edge junctions at orientations spaced every 7.5°. The measurement of the I_c vs θ, which probes mainly the magnitude of the order parameter has an angular anisotropy, indicating an anisotropic order parameter. Also the execution of these experiments is not easy due to the difficulty in cleaning, polishing a crystal at angle θ, between 0 and $\pi/2$.

Also in an experiment analogous to the corner junction Miller *et al.*[22] used frustrated thin-film tricrystal samples to probe the pairing symmetry of YBCO. They found a minimum in the I_c vs the externally applied flux Φ_e diagram at $\Phi_e=0$ in the short junction limit and a maximum at $\Phi_e=0$ for a wide junction where the junction length is much larger than the λ_J. However for a wide junction the correct quantity to be compared should be the total flux Φ which involves contribution both from the externally applied flux and the intrinsic flux. In addition, the tricrystal magnetometry experiments can only observe spontaneous magnetization only for the wide junction limit in the frustrated geometry.[23]

There are a number of complicating factors in the interpretation of the experiments involving corner junctions that could lead to an asymmetric (I_c vs Φ) pattern even for $\theta=0$. These are the asymmetry of the junction (meaning that the critical current of the two junction faces are not equal). This will only cause the dip to be shallower and will maintain the symmetry of the I_c vs Φ diagram. Also these experiments are influenced by the sample geometry and the effect of flux trapping, i.e., there can be vortices trapped between the planes of the cuprate superconductors that could affect the I_c vs Φ diagram. In the corner junction case, it creates an asymmetry in the flux modulation curves. However these flux trapping effects are not sufficiently large to change the qualitative interpretation of these experiments.

VII. CONCLUSIONS

We studied numerically the possible spontaneous vortex states that may exist in a corner junction between a superconductor with time reversal symmetry broken (i.e., $d_{x^2-y^2}+id_{xy}$ or $d_{x^2-y^2}+is$) and an *s*-wave superconductor, in the long junction limit. We studied separately three parameters which can be used to modulate the spontaneous flux. These are the magnetic field H, the interface orientation θ, and the magnitude of the subdominant order parameter n_s. We pointed out the differences between time reversal broken states under these modulation parameters.

We found that in flux modulation experiments involving superconductors with some spontaneous flux the range in magnetic flux of each mode is displaced compared to the case of a pure *s*-wave superconductor junction by an amount which corresponds to the intrinsic flux. In particular when the magnetic field H is considered as the modulation parameter, the range in H of the lower fluxon modes is significantly broadened compared to the *s*-wave case, and the instability at the boundary values of the field sets in due to the interaction of the flux entering from the junction edges with the intrinsic flux. In any case, for each value of the parameter which changes the flux, the modes are separated by a single flux quantum.

We also derived some simple arguments to discriminate between the different pairing states that break the time reversal symmetry. For the $d_{x^2-y^2}+id_{xy}$-wave pairing state, the junction orientation where $\theta=0$, i.e., the lobes of the dominant $d_{x^2-y^2}$-wave order parameter are at right angles for the corner junction, give flux quantization condition $\Phi=n\Phi_0/2$ as in the $d_{x^2-y^2}$-wave state, which is different from the corresponding flux quantization for the $d_{x^2-y^2}+is$-wave pairing state, at $\theta=0$, which is $\Phi=(n/2+f)\Phi_0$, where f is a small quantity. These different conditions provide a way experimentally to distinguish between time reversal broken symmetry states. Note that since the magnitude of the secondary order parameter is small compared to the dominant, the detection of time reversal broken states requires a very precise measurement of the spontaneous magnetic flux.

Also we showed that the magnetic interference pattern at $\theta=0$ is symmetric (asymmetric) for the $d_{x^2-y^2}+id_{xy}$ ($d_{x^2-y^2}+is$), and this also can be used to probe which symmetry the order parameter has, at least where the junctions are formed. We expect our findings, for the magnetic field dependence of the critical current, to hold even in the short junction limit, where the most experiments on corner junctions have been performed.[2,12]

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