

Tunneling conductance in normal metal–triplet superconductor junction

N. Stefanakis*

Department of Physics, University of Crete, P.O. Box 2208, GR-71003 Heraklion, Crete, Greece

Abstract

We calculate the tunneling conductance spectra of a normal metal/insulator/triplet superconductor from the reflection amplitudes using the Blonder–Tinkham–Klapwijk formula. For the triplet superconductor we assume one special p-wave order parameter having line nodes and two two-dimensional f-wave order parameters with line nodes breaking the time-reversal symmetry. Also we examine nodeless pairing potentials. The tunneling peaks are due to the formation of bound states for each surface orientation at discrete quasiparticles trajectory angles. The tunneling spectra can be used to distinguish the possible candidate pairing states of the superconductor Sr_2RuO_4 . © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Tunneling conductance; Metal/insulator/triplet superconductor; Nodeless pairing potential

1. Introduction

The recent discovery of superconductivity in Sr_2RuO_4 has attracted much theoretical and experimental interest [1]. Muon spin rotation experiments show that the time-reversal symmetry is broken for the superconductor Sr_2RuO_4 [2]. Knight-shift measurements show no change when passing through the superconducting state and is a clear evidence for spin triplet pairing state [3]. Also specific heat measurements support the scenario of line nodes within the gap as in the high T_c cuprate superconductors [4].

In tunneling experiments involving singlet superconductors both line nodes and time-reversal

symmetry breaking can be detected from the V-like shape of the spectra and the splitting of the zero energy conductance peak (ZEP) at low temperatures respectively [5–8].

Also the properties of ferromagnet–insulator–superconductor with triplet pairing symmetry have been analysed [9]. It is found that the bound states are suppressed and hence the tunneling conductance peaks are eliminated with the increase of the exchange field.

In this paper we will use the Bogoliubov–de Gennes (BdG) equations to calculate the tunneling conductance of normal metal/triplet superconductor contacts, with a barrier of arbitrary strength between them, in terms of the probability amplitudes of Andreev and normal reflection. For the triplet superconductor we shall assume three possible pairing states of two-dimensional order parameter, having line nodes within the RuO_2

* Tel.: +30-81-394164; fax: +30-81-394101.

E-mail address: stefan@iesl.forth.gr (N. Stefanakis).

plane, which break the time-reversal symmetry. The first two are the 2D f-wave states proposed by Hasegawa et al. [10] having $\mathbf{B}_{1g} \times \mathbf{E}_u$ and $\mathbf{B}_{2g} \times \mathbf{E}_u$ symmetry respectively. The other one is called nodal p-wave state and has been proposed by Dahm et al. [11]. This pairing symmetry has nodes as in the $\mathbf{B}_{2g} \times \mathbf{E}_u$ case. Also we will consider the nodeless p-wave pairing state proposed by Miyake and Narikiyo [12].

2. Theory for the tunneling effect

We consider the normal metal/insulator/superconductor junction shown in Fig. 1. The geometry of the problem has the following limitations. The particles move in the xy -plane and the boundary between the normal metal ($x < 0$) and superconductor ($x > 0$) is the yz -plane at $x = 0$. The insulator is modeled by a delta function, located at $x = 0$, of the form $V\delta(x)$. The temperature is fixed to 0 K. We take the pair potential as a step function i.e. $\Delta_{s\bar{s}}(\mathbf{k}, \mathbf{r}) = \Theta(x)\Delta_{s\bar{s}}(\theta)$, where $k_x, k_y = \cos\theta, \sin\theta$, and $s, \bar{s}$ are spin indices.

Suppose that an electron is incident from the normal metal to the insulator with an angle θ . The electron (hole) like quasiparticle will experience different pair potentials $\Delta_{s\bar{s}}(\theta)(\Delta_{s\bar{s}}(\pi - \theta))$. The amplitudes for Andreev and normal reflection are obtained by solving the BdG equations. Using the

matching conditions of the wave function at $x = 0$, $\Psi_I(0) = \Psi_{II}(0)$ and $\Psi'_{II}(0) - \Psi'_I(0) = (2mV/\hbar^2) \times \Psi_I(0)$, the Andreev and normal reflection amplitudes $R_a = |a|^2$, $R_b = |b|^2$ are obtained

$$R_a = \frac{\sigma_N^2 |n_+|^2}{|1 + (\sigma_N - 1)n_+ n_- \phi_- \phi_+^*|^2}, \quad (1)$$

$$R_b = \frac{(1 - \sigma_N)|1 - n_+ n_- \phi_- \phi_+^*|^2}{|1 + (\sigma_N - 1)n_+ n_- \phi_- \phi_+^*|^2}. \quad (2)$$

The BCS coherence factors are given by

$$u_{\pm}^2 = \left[1 + \sqrt{E^2 - |\Delta_{\pm}(\theta)|^2} / E \right] / 2, \quad (3)$$

$$v_{\pm}^2 = \left[1 - \sqrt{E^2 - |\Delta_{\pm}(\theta)|^2} / E \right] / 2, \quad (4)$$

and $n_{\pm} = v_{\pm}/u_{\pm}$. The internal phase coming from the energy gap is given by $\phi_{\pm} = [\Delta_{\pm}(\theta)/|\Delta_{\pm}(\theta)|]$, where $\Delta_+(\theta) = \Delta(\theta)$ ($\Delta_-(\theta) = \Delta(\pi - \theta)$), is the pair potential experienced by the transmitted electron-like (hole-like) quasiparticle. $\Delta(\theta) = \Delta_{\uparrow\downarrow}(\theta) = \Delta_{\downarrow\uparrow}(\theta)$, since the Cooper pairs have zero spin projection i.e. $\mathbf{d} \parallel \hat{\mathbf{z}}$.

The tunneling conductance, normalized by that in the normal state is given by [5]

$$\sigma(E) = \frac{\int_{-\pi/2}^{\pi/2} d\theta (\bar{\sigma}_{\uparrow}(E, \theta) + \bar{\sigma}_{\downarrow}(E, \theta))}{\int_{-\pi/2}^{\pi/2} d\theta 2\sigma_N}. \quad (5)$$

According to the Blonder–Tinkham–Klapwijk (BTK) formula the conductance of the junction $\bar{\sigma}_s(E, \theta)$, for spin $s = \uparrow, \downarrow$, is expressed in terms of the probability amplitudes a , and b as

$$\bar{\sigma}_s(E, \theta) = 1 + R_a - R_b. \quad (6)$$

The transparency of the junction σ_N is connected to the barrier height V by the relation

$$\sigma_N = \frac{4 \cos^2 \theta}{Z^2 + 4 \cos^2 \theta}, \quad (7)$$

where $Z = 2mV/\hbar^2 k_F$, denotes the strength of the barrier.

The pairing potential is described by a 2×2 form

$$\hat{\Delta}_{s\bar{s}}(\mathbf{k}) = \begin{pmatrix} -d_x(\mathbf{k}) + id_y(\mathbf{k}) & d_z(\mathbf{k}) \\ d_z(\mathbf{k}) & d_x(\mathbf{k}) + id_y(\mathbf{k}) \end{pmatrix}, \quad (8)$$

in terms of the $d(\mathbf{k}) = (d_x(\mathbf{k}), d_y(\mathbf{k}), d_z(\mathbf{k}))$ vector.

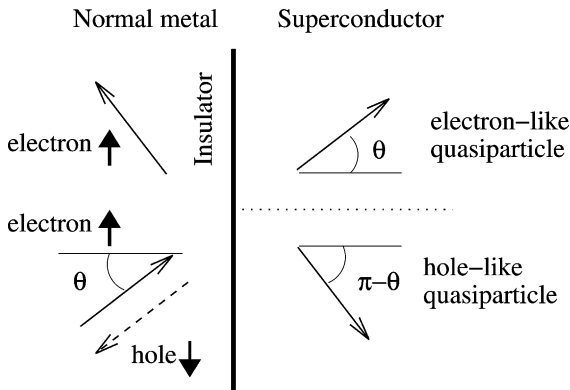


Fig. 1. The geometry of the normal metal/insulator/superconductor interface. The vertical line along the y -axis represents the insulator. The arrows illustrate the transmission and reflection processes at the interface. θ is the angle of the incident electron beam and the normal.

We consider the following pairing symmetries for Sr_2RuO_4 .

- (a) In the first 2D f-wave state $\mathbf{B}_{1g} \times \mathbf{E}_u$ $d_z(\mathbf{k}) = \Delta_0(k_x^2 - k_y^2)(k_x + ik_y)$. This state has nodes at the same points as in the $d_{x^2-y^2}$ -wave case.
- (b) For the second 2D f-wave state $\mathbf{B}_{2g} \times \mathbf{E}_u$ $d_z(\mathbf{k}) = \Delta_0 k_x k_y (k_x + ik_y)$. This state has nodes at $0, \pi/2, \pi, 3\pi/2$ and has also been studied by Graf and Balatsky [13].
- (c) In case of a nodal p-wave superconductor $d_z(\mathbf{k}) = \Delta_0/s_M[\sin(k_x a) + i \sin(k_y a)]$, with $k_x a = R\pi \cos(\theta - \beta)$ and $k_y a = R\pi \sin(\theta - \beta)$, $s_M = \sqrt{2} \sin(\pi/\sqrt{2}) = 1.125$, and $R = 1$ in order to have a node in $\Delta(\theta)$ [11]. This state has nodes as in the $\mathbf{B}_{2g} \times \mathbf{E}_u$ state.
- (d) In case of a nodeless p-wave superconductor, proposed by Miyake and Narikiyo [12] $d_z(\mathbf{k}) = \Delta_0/s_M[\sin(k_x a) + i \sin(k_y a)]$, with $k_x a = R\pi \cos(\theta - \beta)$ and $k_y a = R\pi \sin(\theta - \beta)$, $s_M = \sqrt{2} \sin(\pi/\sqrt{2}) = 1.125$, and $R = 0.9$. This state does not have nodes.

3. Results

In Fig. 2 we plot the tunneling conductance $\sigma(E)$ as a function of E/Δ_0 for $Z = 10$, for different orientations (a) $\beta = 0$, (b) $\pi/8$, (c) $\pi/4$. The pairing symmetry of the superconductor is $\mathbf{B}_{1g} \times \mathbf{E}_u$ in Fig. 2(a), $\mathbf{B}_{2g} \times \mathbf{E}_u$ in Fig. 2(b), nodal p-wave, in Fig. 2(c), nodeless p-wave in Fig. 2(d). The temperature is fixed to 0 K. In all the cases we see the presence of a large residual density of states within the energy gap and peaks due to the sign change of the pair potential at discrete values of θ , for fixed β . Also the linear increase with E is consistent with the presence on line nodes in the pairing potential.

Generally the spectra for the three pairing symmetries with line nodes depends strongly on the position of the nodes in the pairing potential and the orientation angle β . The spectra for angle β in the $\mathbf{B}_{1g} \times \mathbf{E}_u$ seen in Fig. 2(a) case is identical to the spectra of $\mathbf{B}_{2g} \times \mathbf{E}_u$ in Fig. 2(b) for angle $(\pi/4) - \beta$, since the node positions for the two symmetries differ by $\pi/4$. The nodal p-wave case in

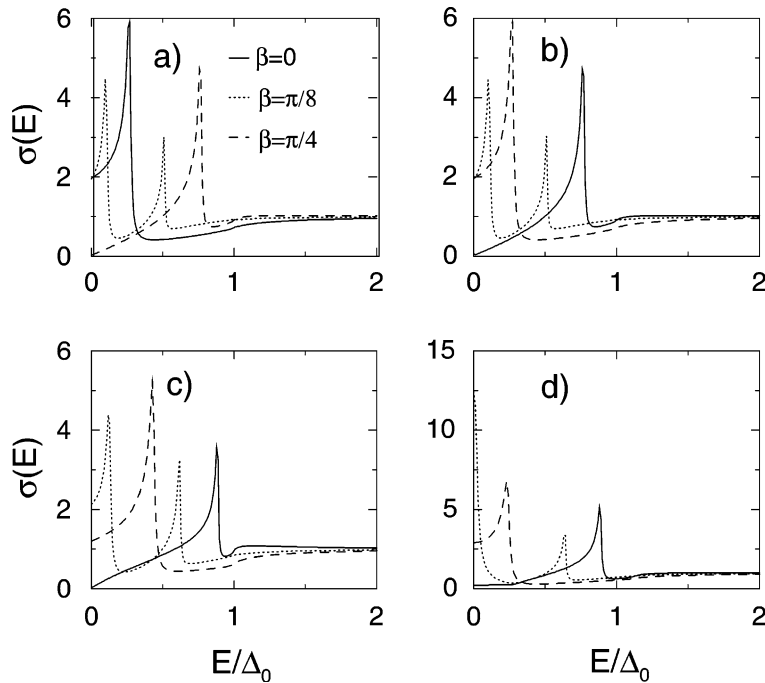


Fig. 2. Normalized tunneling conductance $\sigma(E)$ as a function of E/Δ_0 for different orientations $\beta = 0$ (—), $\pi/8$ (···), $\pi/4$ (- -). The temperature is $T = 0$, $Z = 10$. The pairing symmetry of the superconductor is (a) $\mathbf{B}_{1g} \times \mathbf{E}_u$, (b) $\mathbf{B}_{2g} \times \mathbf{E}_u$, (c) nodal p-wave and (d) nodeless p-wave.

Fig. 2(c) has the same nodal structure as the $B_{2g} \times E_u$ case and we see that the spectra for these two candidates are similar. For nodeless pairing states a subgap or a full gap opens in the tunneling spectra. This is seen in Fig. 2(d), for the nodeless p-wave pairing state.

The peaks in the tunneling conductance are explained from the formation of bound states within the gap. The bound state energies E_p , are given from the values of E where the denominator of Eqs. (1) and (2) vanishes. In this case the Andreev reflection coefficient is equal to unity, and the effect of the boundary is turned off. The corresponding equation is written as [7]

$$\phi_- \phi_+^* n_+ n_-|_{E=E_p} = 1.0. \quad (9)$$

Bound states are formed because the transmitted electron-like and hole-like quasiparticles feel different sign of the pairing potential. These bound states occur for a given orientation β at discrete values of θ , as seen in Fig. 3 where the bound state energy E_p is plotted for $\beta = 0, \pi/8, \pi/4$ as a function of θ for the various pairing states. For a given value of β , the conductance $\bar{\sigma}(E, \theta) = 2$ at the angles θ where bound state occurs and the tunneling

conductance $\sigma(E)$ is enhanced due to the normal state conductance σ_N . The residual values of the tunneling conductance within the gap are due to the bound states and the peaks are formed at energies where the number of bound states is increased. As seen in Fig. 3(a) for the pairing state $B_{1g} \times E_u$ for $\beta = 0$, at the energy in the interval $0.27 < E < 1$ only one bound state occurs. At $E = 0.27$ two more bound states are formed and the peak seen in Fig. 2(a) is due to these new bound states. The same happens for $\beta = \pi/8$ at $E = 0.1$ and $E = 0.5$, where an increased number of bound states is formed, as seen in Fig. 2(a) (dotted line). Also for $\beta = \pi/4$ in Fig. 3(a) (dashed line) the conductance peak is formed for energy close to $E = 0.8$, where two bound states are formed. In the $B_{2g} \times E_u$ pairing state the bound states and also the position of the peaks for an angle β is the same as in the $B_{1g} \times E_u$ pairing state for the angle $(\pi/4) - \beta$. In the nodal p-wave state seen in Fig. 3(c) the bound states are symmetric to the $B_{2g} \times E_u$ case with respect to $\theta = 0$. However this does not influence the energy levels which occur almost at the same position as in the $B_{2g} \times E_u$ -wave case. In the nodeless p-wave pairing state, for $\beta = \pi/8$ close to $E = 0$ we have a

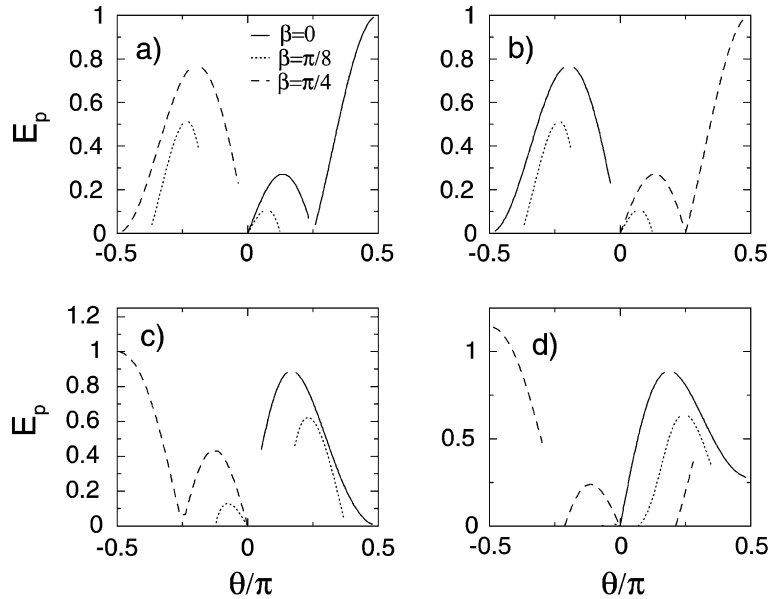


Fig. 3. Bound state energy E_p (in units of Δ_0) for $\beta = 0, \pi/8, \pi/4$ as a function of θ . The temperature is $T = 0$. The pairing symmetry of the superconductor is (a) $B_{1g} \times E_u$, (b) $B_{2g} \times E_u$, (c) nodal p-wave and (d) nodeless p-wave.

pair of new bound states and the peak seen in Fig. 2(d) is due to these new bound states.

We calculated the tunneling conductance in normal metal/insulator/triplet superconductor using the BTK formalism. We assumed nodal and nodeless pairing potentials, breaking the time-reversal symmetry. The residual values are due to the formation of bound states for each β at discrete values of the angle θ for energies within the gap. The peaks occur at the energies where an increasing number of bound states is formed. Thus the spectra shows a double peak structure, for the low transparency limit, for all the pairing states described in this paper. This conclusion is in agreement with recent analytical calculation [14].

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