Effect of magnetic field in superconductor hybrid interfaces

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Abstract
We examine the effect of the magnetic field on the proximity effect in nanostructures self-consistently using the Bogoliubov–de Gennes formalism within the two-dimensional extended Hubbard model. We calculate the local density of states, and the pair amplitude. We study several nanostructures: superconductor–two-dimensional electron gas, superconductor–ferromagnet. In these structures the magnetic field can be considered as a modulation parameter for the proximity effect.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

When a magnetic field is applied perpendicularly in a two-dimensional electron gas (2DEG) it leads to a fractal energy spectrum known as the Hofstadter butterfly [1]. Very recently it was possible to probe the Hofstadter spectrum in 2DEG structures with a weak lateral superlattice potential having a square symmetry [2]. In the minigaps of the spectrum the Hall conductance was found to be quantized in integer multiples of e^2/h [2].

In the same regime of high magnetic fields the hybrid system of superconductor and a 2DEG has interesting properties as well. Recently conductance measurements have been performed on a superconductor–2DEG structure in a perpendicular external magnetic field [3]. In this case the magnetic field is sufficiently large to induce Landau quantization in 2DEG. However it is smaller than the upper critical field of the superconductor. It was found that due to modified Andreev reflection the resistance decreases with the magnetic field.

The hybrid structures composed of superconductors and metals or ferromagnets provide ideal systems to probe the pairing symmetry of high-T_c superconductors [4, 5]. Conductance experiments report the existence of a zero-bias conduction peak (ZBCP) [6] which originates from the zero-energy states (ZES) formed near the [110] surfaces of d-wave superconductors [7, 8].

Dealing with hybrid systems, a phenomenon which is of great importance is the proximity effect namely the penetration of the superconducting correlations in a metal which is not a superconductor. Recently the proximity effect has been probed as decaying oscillations of the density of states in s-wave superconductor–ferromagnet hybrid structures [9] and a phase shift of half flux quantum in the diffraction pattern of a ferromagnetic 0–π SQUID [10]. Similar effects have been observed in d-wave [11, 12] superconductor–ferromagnet hybrid structures. Theoretical explanation has been given in the framework of the quasiclassical theory for s-wave [13] and d-wave cases [14]. In these structures the exchange field modulates the period of the pair amplitude oscillations. Moreover much interest has been recently focused on the manipulation of entangled states which are formed by extracting Cooper pairs from the superconductor. For example a beam splitter has been proposed [15] and also several experiments that involve ferromagnetic electrodes connected to superconductors [16–18]. These structures have acquired considerable interest in the last few years due to the possibility of using the π states in solid-state qubit implementation.

In this paper our goal is to explore several aspects related to the control of the proximity effect in superconductor–2DEG and superconductor–ferromagnet in the presence of strong magnetic fields. We assume that the superconductor has high critical field, so that the applied magnetic field is restricted to the 2DEG region. The basic quantities which we calculate are the local density of states (LDOS) and the pair amplitude as a function of several relevant parameters: the distance from the surface, the magnetic field, the barrier strength and the symmetry of the pair potential. The method is based on exact diagonalizations of the Bogoliubov–de Gennes

(BdG) equations associated with the mean-field solution of an extended Hubbard model.

Our principal result is that for superconductor–2DEG interface the LDOS shows a composite picture of energy bands due to Landau quantization and gaps due to the presence of superconductivity. In superconductor–ferromagnet hybrid structures the magnetic field can modulate the period of the pair amplitude oscillations inside the ferromagnetic layer. We propose an experiment for d-wave superconductors–2DEG junction in the QHE regime.

This paper is organized as follows. In section 2 we develop the model and discuss the formalism. In section 3 we discuss the effect of the magnetic field in pure 2DEG. In section 4 we discuss the effect of the magnetic field and the effect of the strength of the barrier, in superconductor–2DEG heterostructures. In section 5 we discuss the effect of the magnetic field in a superconductor–ferromagnet hybrid structure. In section 6 we discuss the relevance to experiments. Finally a summary and discussion are presented in the last section.

2. Formalism

The Hamiltonian of the extended Hubbard model on a two-dimensional square lattice takes the form

$$H = -\sum_{\langle i,j\rangle} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + \sum_{i\sigma} \mu^{I}_{i} n_{i\sigma} + \sum_{i\sigma} h_{i\sigma} n_{i\sigma} + V_0 \sum_{i} n_{i\uparrow} n_{i\downarrow} + \frac{V_1}{2} \sum_{\langle ij\rangle\sigma\sigma'} n_{i\sigma} n_{j\sigma'}, \tag{1}$$

where i, j are site indices and the angle brackets indicate that the hopping is only to nearest neighbours. h is the exchange field, and μ^{I}_{i} is the interface potential. $n_{i\sigma} = c^{\dagger}_{i\sigma} c_{i\sigma}$ is the electron number operator at site i. V_0, V_1 are on site and nearest-neighbour interaction strength. Both are assumed equal to $-2.5t$. In the presence of a magnetic field the hopping integral t_{ij} is modified by the Peierls substitution, which accounts for the coupling of electrons to the magnetic field

$$t_{ij}(A) = t_{ij} \exp\left(\frac{\mathrm{i}e}{\hbar c} \int_{r_j}^{r_i} A\, \mathrm{d}l \right). \tag{2}$$

A is the vector potential which is chosen in the Landau gauge $A = (-By, 0, 0)$, where B is the magnetic field applied perpendicular to the lattice in the z-direction. We define the dimensionless magnetic flux f as follows: $2\pi f = e/c\hbar Ba^2 = 2\pi\Phi/\Phi_0$, where a is the lattice constant, Φ is the flux per plaquete, Ba^2, and $\Phi_0 = ch/e$ is the flux quantum.

Within the mean-field approximation equation (1) reduces to the BdG equations [19]. We solve numerically the BdG equations. The numerical procedure can be found in our previous publications [7, 18]. Within the present work we are interested in the calculation of the proximity induced pair amplitude and the local density of states in superconductor–2DEG junctions in the presence of a strong magnetic field.

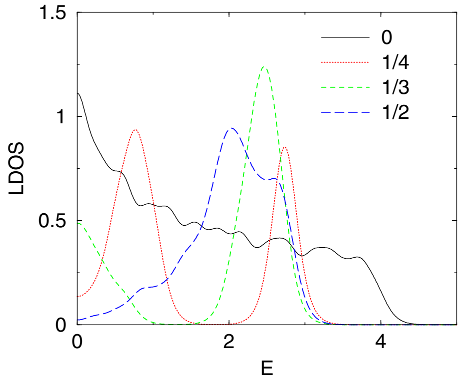

Figure 1. The LDOS as a function of energy, for a 2DEG, for different values of the magnetic field $f = 0, 1/4, 1/3, 1/2$.

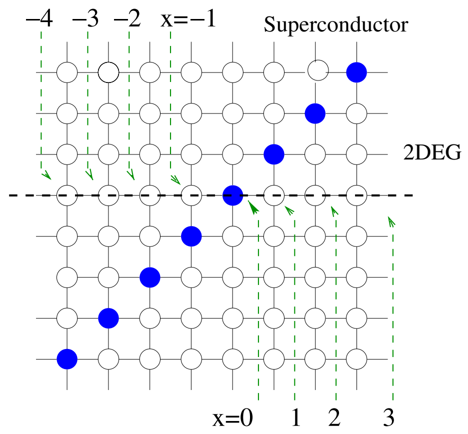

Figure 2. The superconductor–2DEG interface. The solid circles indicate interface sites. The LDOS is presented for sites along the dashed line. The 2DEG extends to the right of the interface.

3. 2DEG

We start our investigation of heterostructures by discussing the effect of the magnetic field on the local density of states for a 2DEG lattice. We consider a two-dimensional system of 20×20 sites, and we suppose fixed boundary conditions by setting the impurity potential $\mu^{I} = 100t$ at the surface. The temperature is $k_B T = 0.1t$. In order to achieve the values of the magnetic flux close to unit, the lattice constant must be tuned to a large value of the order of μm. This can be experimentally achieved by using artificial superlattices with controlled lattice constant as we discuss later in section 6.

The LDOS as a function of energy, for a two 2DEG, for different values of the magnetic flux is shown in figure 1 (we show only the $E > 0$ part of the spectrum due to symmetry). We see that when the magnetic flux through a plaquette is a rational number p/q the energy spectrum has q sub-bands. At each half-filled sub-band there is a logarithmic singularity. The spectrum obtained corresponds to the Hofstadter butterfly.

4. Superconductor–2DEG

We now discuss the modification of the Hofstadter spectrum in a superconductor–2DEG heterostructure as shown in figure 2 due to the magnetic field. We choose the interface in the

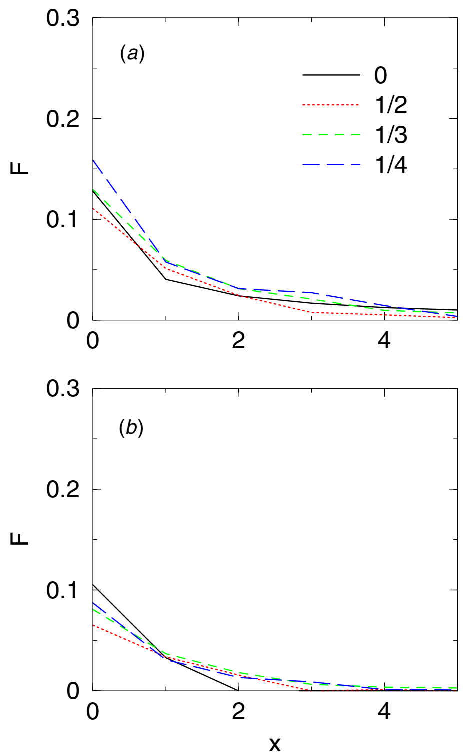

Figure 3. (*a*) The pairing amplitude as a function of position, for a s-wave superconductor–2DEG interface, for different values of the magnetic field $f = 0, 1/4, 1/3, 1/2$. (*b*) The same but for a d-wave superconductor.

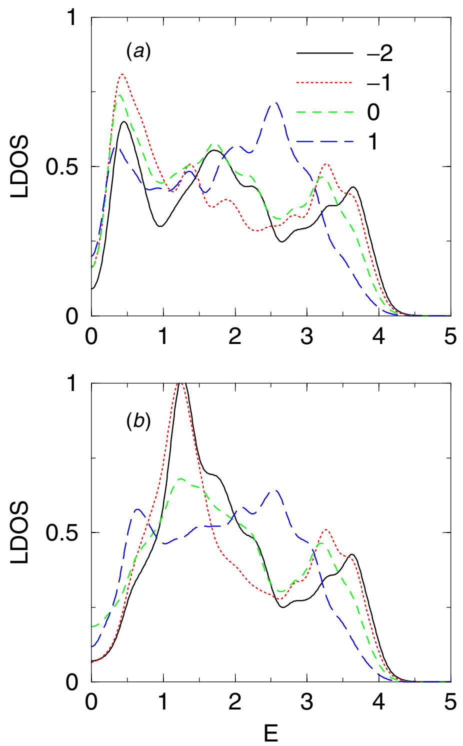

Figure 4. (*a*) The LDOS as a function of energy, for an s-wave superconductor–2DEG interface, for different values of distance from the interface $x = -2, -1, 0, 1$, for a magnetic field equal to $f = 1/2$. (*b*) The same but for a d-wave superconductor.

[110] direction along the nodes of the d-wave pair potential. For this direction modifications are expected in the LDOS for the d-wave pairing state compared to the s-wave state due to the phase of the order parameter. We assume that the superconductor has high critical field, so in the calculation the magnetic field is restricted to the 2DEG region. When the magnetic field is absent, the pair potential decays exponentially inside the 2DEG for both s-wave and d-wave superconductors (see figure 3). As we increase the magnetic field the pair amplitude is not really modified inside the 2DEG. However inside the superconductor the pair amplitude increases for s-wave and does not change for d-wave.

The proximity effect is even more obvious in the LDOS (see figure 4). We see that for a magnetic field equal to $f = 1/2$, the site inside the superconductor ($x = -2$) shows a gap structure as a signature of superconductivity and also a peak at $E = 2$ which is due to the effect of the magnetic field. As we move to the interior of the 2DEG the gap disappears while the peak at $E = 2$ becomes more pronounced. The effect is similar to the s-wave and the d-wave cases. So we may conclude that the Hofstadter spectrum can be observed even in the absence of an external magnetic field, but only due to the proximity effect with a superconductor, and also that the same spectrum is strongly modified in the presence of superconducting correlations.

In general, the increase of the inter-facial barrier potential suppresses the proximity effect because the leakage of Cooper pairs from the superconductor to the 2DEG is reduced if the tunnel amplitude is reduced. In this case the magnetic field is screened by the barrier. As a consequence for both s-wave and d-wave the pair amplitude is reduced on the 2DEG side by the increase of the barrier strength as seen in figure 5. In the s-wave case (see figure 5(*a*)) the pair amplitude on the superconducting side increases by the increase of the barrier strength since the barrier decouples the superconductor from the region where the magnetic field is applied, and the influence of the magnetic field in the superconductor is of reduced strength. In the d-wave case on the other hand the pair amplitude close to the interface does not change much, since the interface becomes pair breaking.

5. Superconductor–ferromagnet

We demonstrate in this section that the magnetic field can act as a control parameter for the proximity effect in hybrid structures. We study first the effect of the magnetic field in a pure ferromagnetic state. Because of the presence of the exchange field in the ferromagnet the energy bands in the LDOS that are formed due to the presence of the magnetic field are spit (see figure 6).

In hybrid structures in the absence of magnetic field, there exist oscillations of the pair amplitude in the ferromagnetic layer. The magnetic field changes the period of the oscillations of the pair amplitude as seen in figure 7. This happens because the magnetic field favours single spin orientation and therefore modifies the exchange field of the ferromagnet to a different effective exchange field. When the magnetic field increases the effective exchange field increases and the period of oscillations of the pair amplitude decreases as seen in figure 7.

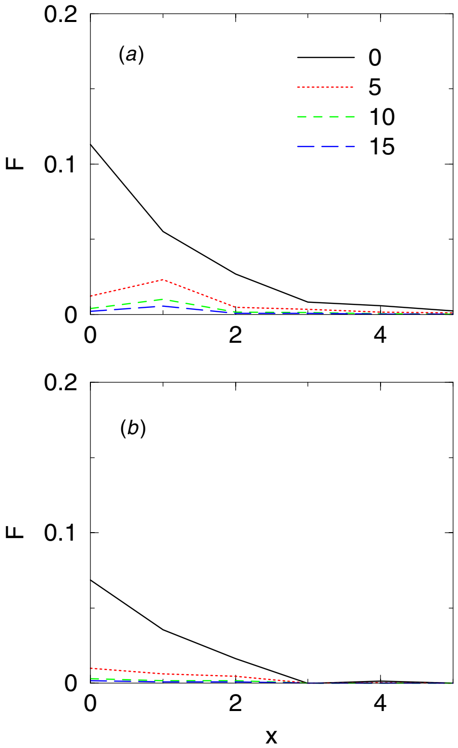

Figure 5. (*a*) The pair amplitude as a function of distance from the interface, for a s-wave superconductor–2DEG interface, for different values of barrier strength 0, 5, 10, 15, for a magnetic field equal to $f = 1/2$. (*b*) The same but for a d-wave superconductor.

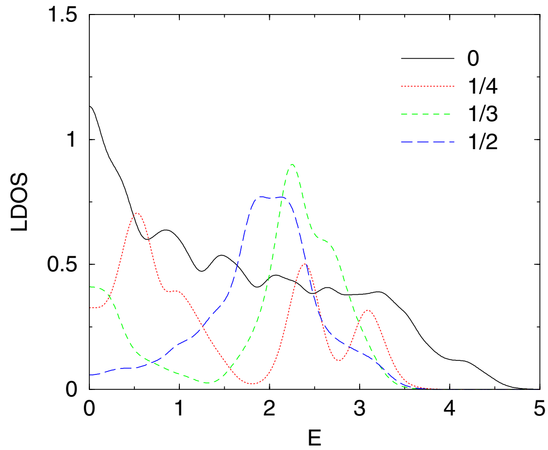

Figure 6. The LDOS as a function of energy, for a 2DEG, for different values of the magnetic field $f = 0, 1/4, 1/3, 1/2$, for an exchange field equal to $h = 0.5$.

The proximity effect in hybrid structures appears also as oscillations in the local density of states with the distance from the interface. The external magnetic field modifies the period of these oscillations. We can see the effect of the magnetic field in the LDOS in figure 8. We see that for $f = 0$ the peak in the LDOS which is a signature of the π state appears after the $x = 3$ site from the interface, while for $f = 1/2$ it appears for smaller distance, because the effective period of the oscillations has been changed due to the presence of the magnetic field.

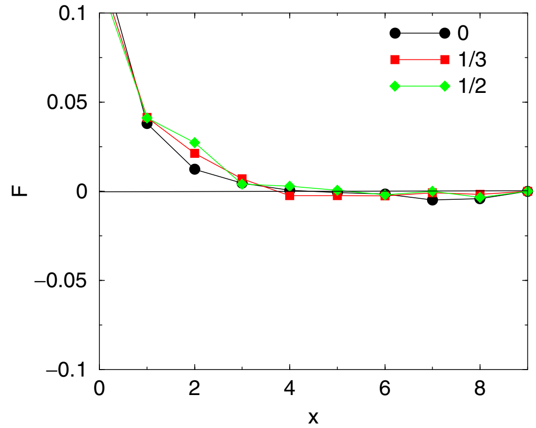

Figure 7. The pairing amplitude as a function of position, for a superconductor–2DEG interface, for different values of the magnetic field $f = 0, 1/3, 1/2$, for exchange field equal to $h = 0.5$.

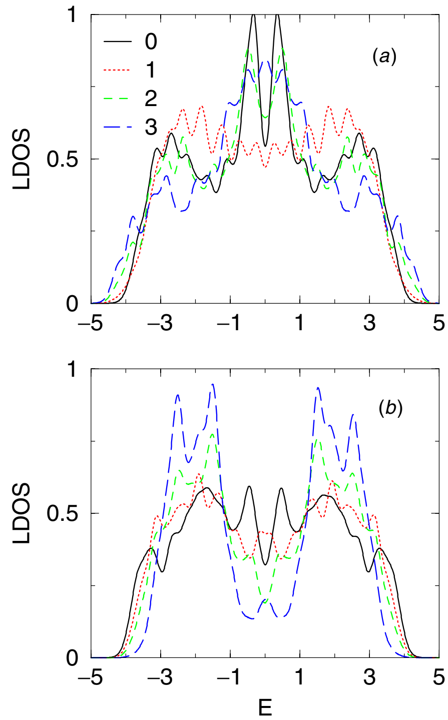

Figure 8. (*a*) The LDOS as a function of energy, for a superconductor–2DEG interface, for different values of the distance from the interface $x = 0, 1, 2, 3$ for exchange field equal to $h = 0.5$, and magnetic field $f = 0$. (*b*) The same as in (*a*) but for $f = 1/2$.

6. Relevance to experiment

We comment mainly in this section on the effect of the discrete character of the model and we state to what extend the results are valid more generally and can be observed in experiments. Let us comment first on the possibility of observing the coexistence of superconducting gap and Hofstadter spectrum in real systems. Assuming a lattice constant of the order of 2Å, the magnetic field required to obtain $f = 1$ is quite high of the order of 10^5 T. We propose here an alternative way to overcome this problem. It is possible to create lateral two-dimensional artificial superlattices with increased lattice constant patterned on top of the 2DEG [2]. Then for a network cell of the order of 1 μm^2 the magnetic field that corresponds to $f = 1$ is of the order of 1 mT which is experimentally accessible. Within this technique it is possible to observe qualitatively the results presented in the present paper.

Next we comment on the fabrication of the hybrid structures that we mention in the manuscript. Presently using evaporation techniques it is possible to build junctions of normal metals–d-wave superconductor and ferromagnet–d-wave superconductors. For example, in [11, 12] Ag/BSCCO and Fe/Ag/BSCCO junctions were constructed. In addition the junction's orientation can be controlled and set to $\beta = 45$ as it is in our case. In the Fe/Ag/BSCCO junction the conductance is influenced by spin-dependent broken time reversal states and the exchange interaction. In [12] Fe/Ag/BSCCO junctions with the Fe region imparted in layers of equal thickness were prepared in order to study the proximity effect in the Fe region. The conductance showed oscillations in the Fe region as an indication of the proximity effect. It would be useful to test whether these findings are influenced by the application of an additional magnetic field as in the present work.

Finally we propose an experiment for d-wave superconductor–2DEG. It would also be interesting to prepare d-wave–2DEG junctions in the quantum Hall effect (QHE) regime as it has been done for the s-wave case [3] and test the results that we have found. In [3] high mobility GaAs:Al$_x$Ga$_{1-x}$ As heterostructure with high critical field was built, which allows the observation of Andreev reflection up to 3 T in the QHE regime. The main result is a decreased differential resistance at high fields due to the Andreev reflection processes. The same experiment we propose for d-wave superconductor–semiconductor heterostructure. Since high-T_c superconductors have high critical fields it may be possible to construct interfaces that operate in the QHE regime. We expect that in these structures additional effects will appear due to the phase of the d-wave order parameter.

7. Conclusions

We calculated the LDOS and the pair amplitude for several superconductor–2DEG heterostructures, in the presence of an external magnetic field within the extended Hubbard model, self-consistently. In the calculated quantities, such as the pair amplitude and the LDOS, for the sites that are close to the interface, the gap which is a signature of superconductivity coexists with the bands which are formed due to the rational values of the magnetic field. We also demonstrated that the Hofstadter spectrum is strongly modified due to the proximity effect with the superconductor. In superconductor–ferromagnet structures the magnetic field can also be used as an external control parameter in order to modify the proximity effect.

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